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Interfacial coplanar cracks in piezoelectric bi-material systems under pure mechanical impact loading

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Abstract

In this paper, we examine the dynamic behaviour of different piezoelectric bi-material combinations containing two interfacial cracks subjected to mechanical impact loading. The problem is formulated in terms of integral transforms techniques and the collocation method to obtain the solution for the resulting singular integral equation in the transformed plane. Laplace inversion was then used to obtain the resulting dynamic stress intensity factors in the physical plane. Numerical examples are provided for five different types of piezoelectric bi-material systems to illustrate the effect of the presence of collinear interacting cracks and the different material combinations upon the resulting dynamic stress intensity factors.

Keywords: Interfacial crack; Piezoelectric bi-materials; Impact loading; Dynamic stress intensity factor; Interaction

1. Introduction

The study of the fracture mechanics of piezoelectric materials plays an important role in the design of piezoelectric devices. In recent years, a great amount of work has been carried out in this field. However, in spite of the fact that piezoelectric materials and their composites are mostly being used or considered for use in situations involving dynamic loading, most of the existing works deal with the quasi-static treatment of these materials, especially when multiple cracks are involved. This has prompted the undertaking of the present study.

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A number of researchers have contributed to the static response of interfacial cracks presented in piezoelectric composites. For example, Suo et al. (1992) discussed the general treatment of an interfacial crack in an anisotropic piezoelectric composite. They obtained closed form solutions for infinite piezoelectric materials and bi-materials containing a central crack. A new type of singularity is revealed around the interfacial crack tip. By virtue of the principle of analytical continuation and the complex series expansion method, Zhong and Meguid (1997) investigated the interfacial debonding of a circular inhomogeneity in a piezoelectric material. Qin and Yu (1997) studied the arbitrarily oriented inplane crack terminating at the interface of dissimilar piezoelectric materials. As the crack coincided with the interface, the oscillating singularity was recovered.

Dynamic response of cracked piezoelectric materials under dynamic loads are increasingly studied recently (Chen and Karihaloo, 1999; Chen and Meguid, 2000; Wang and Yu, 2001; Li and Tang, 2003, for example). Dynamic interactions between multiple cracks in piezoelectric materials have been investigated by a few researchers in the last few years (see, e.g., Kwon et al., 2002; Meguid and Wang, 1998; Chen and Worswick, 2000; Meguid and Chen, 2001, etc.). However, investigations into the dynamic response of interacting interface cracks in piezoelectric composites are rarely seen in the literature, particularly when transient impacting load is involved.

Chen et al. (1998) investigated the moving interface crack problem in piezoelectric bi-materials and obtained a closed form solution, which indicated the effect of the crack velocity upon the dynamic intensities of stress and electric displacement. Qin and Mai (1999) presented a closed crack tip model for the interface crack in thermo-piezoelectric materials in order to eliminate the physical difficulty of observing oscillating singularity at the crack tip. Using the extended Stroh's formulation and the principle of analytical continuation, Shen and Kuang (1998) solved the interface crack problem in bi-piezothermoelastic media. Narita and Shindo (1998) investigated the interface crack problem in a layered piezoelectric material under antiplane shear loading. The results were obtained in terms of the solution of dual integral equations. Shindo et al. (1998) analyzed the dynamic bending of a symmetric piezoelectric laminated plate with a through crack under harmonic wave loading. The resulting dynamic moment intensity factor was obtained in terms of the solution of the appropriate dual integral equations. As for the multiple-crack problem in piezoelectric materials, there exist some relevant contributions. Pak and Goloubeva (1996), for example, exploited general electro-elastic properties of crack-weakened piezoelectric materials under longitudinal shear loading with the method of Green functions. By employing integral transform technique and a self-consistent iterative technique, Meguid and Wang (1998) examined the dynamic interaction between two cracks in a piezoelectric medium under incident antiplane shear wave loading. Qin and Mai (1998) analyzed the multiple cracks in one side of a thermo-electro-elastic bi-material with the aid of Green function. The interaction of the interface and the cracks upon the stress and electric displacement intensity factors were illustrated by solving appropriate singular integral equations. More recently, Nishioka et al. (2003) obtained the relationships between the dynamic J integral and the stress and electric displacement intensity factors using the near-tip analytical solutions for the interfacial crack in piezoelectric bi-materials. Zhou et al. (2005) explored the dynamic response of two collinear interface cracks in magneto-electro-elastic materials. In their study, the Schmidt method (Morse and Feshbach, 1958; Yan, 1967, for example) was employed to consider the steady-state response of two interface collinear cracks under incident elastic shear wave load. Gu et al. (2002) examined the dynamic transient response of a single interface crack in piezoelectric materials under electro-mechanical impact load.

In the present work, the transient response of two coplanar cracks along the interface of piezoelectric bi-materials is studied under antiplane mechanical impact loading. By employing integral transform techniques, the present mixed boundary value problem is reduced to a Cauchy-type singular integral equation of the first kind. The resulting singular integral equations were solved using the collocation method, developed by Erdogan and Gupta (1972), to provide the dynamic stress intensity factors (DSIF) in the Laplace plane. A numerical Laplace transform inversion technique, described by Miller and Guy (1966), is then used to obtain the DSIF in the physical plane. In order to illustrate the effect of collinear cracks interaction and material combinations upon the impact response of cracked piezoelectric bi-materials, numerical examples are provided for five different combinations commonly used in the design of intelligent structures.

2. Formulation of the problem

Let us consider two coplanar Griffith cracks of the same length lying along the interface of two dissimilar piezoelectric materials. The cracks are located along the x-axis from "$-b$" to "$-a$" and from "a" to "b", using the rectangular coordinate system (x, y, z) shown in Fig. 1. The piezoelectric bi-material is poled in the z-direction, which guarantees its transversely isotropic nature. Let us further assume that the bi-material system is subjected to dynamic impact loading, $\tau_0 H(t)$, applied at infinity.

The present problem can be decomposed into two sub-problems. The first describes a uniform antiplane stress $\tau_0 H(t)$ in a crack-free piezoelectric bi-material, while the second takes into account the antiplane stress $-\tau_0 H(t)$ acting on the surfaces of the two coplanar cracks. Sub-problem (b) is of interest to the current study.

In this case, the boundary tractions acting on the crack surfaces are described by

$$\tau_{zy}^{(1)}(x,0,t) = \tau_{zy}^{(2)}(x,0,t) = -\tau_0 H(t), \quad a < |x| < b \tag{1}$$

subject to the following continuity condition along the interface of the piezoelectric bi-material

$$\left.\begin{aligned} w^{(1)}(x,0,t) &= w^{(2)}(x,0,t) \\ \tau_{zy}^{(1)}(x,0+,t) &= \tau_{zy}^{(2)}(x,0-,t) \end{aligned}\right\} \quad 0 < |x| < a, \quad |x| > b \tag{2}$$

where $\tau_{kz}^{(i)}$ $(k = x, y)$ represent the antiplane stress and $w^{(i)}$ the mechanical displacement. The superscripts "i" $(i = 1, 2)$ represents the respective quantities in the upper and lower half spaces and this convention holds throughout the paper.

The electric boundary conditions of a crack in piezoelectric media had been the topic of many investigations (McMeeking, 1989; Hao and Shen, 1994; Dunn, 1994). Liu and Chen (2002), and Wang and Mai (2004) compared the permeable and impermeable assumption respectively using simple crack problem.

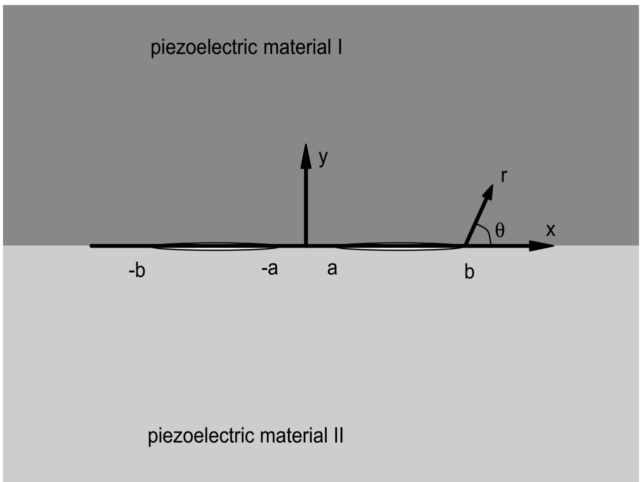

Fig. 1. A schematic of a piezoelectric bi-material system containing two coplanar interfacial cracks under antiplane impact loading.

Recently, Yang (2001) and Wang and Zhang (2004) employed the electric field saturation model developed by Gao et al. (1997) to investigate the mode I fracture behaviour of piezoelectric materials.

For the present study, since no opening displacement exists, the crack faces can therefore be assumed to be in perfect contact. Accordingly, the electric permeable condition will be enforced. That is, both the electric potential and the normal electric displacement are assumed to be continuous across the interfacial cracks, which can be expressed as

$$\phi^{(1)}(x,0+,t)=\phi^{(2)}(x,0,t),\quad |x|<\infty \tag{3}$$

$$D_y^{(1)}(x,0+,t)=D_y^{(2)}(x,0-,t),\quad |x|<\infty \tag{4}$$

where $D_k^{(i)}$ $(k=x,y)$ represent the electric displacement vector, $\phi^{(i)}$ the electric potential. The equilibrium and the Maxwell equations for piezoelectric under antiplane loading can be expressed as follows:

$$\partial\tau_{xz}^{(i)}/\partial x+\partial\tau_{yz}^{(i)}/\partial y=\rho_i\partial^2 w^{(i)}/\partial t^2 \tag{5}$$

$$\partial D_x^{(i)}/\partial x+\partial D_y^{(i)}/\partial y=0 \tag{6}$$

with ρ_i, $(i=1,2)$ being the mass density of piezoelectric ceramics.

The antiplane constitutive equations for transversely isotropic piezoelectric materials can be expressed by

$$\tau_{zj}^{(i)}=c_{44}^{(i)}w_{,j}^{(i)}+e_{15}^{(i)}\phi_{,j}^{(i)} \tag{7}$$

$$D_j^{(i)}=e_{15}^{(i)}w_{,j}^{(i)}-\varepsilon_{11}^{(i)}\phi_{,j}^{(i)} \tag{8}$$

where $c_{44}^{(i)}$, $e_{15}^{(i)}$, and $\varepsilon_{11}^{(i)}$ are the shear modulus, piezoelectric coefficient and dielectric parameter of the piezoelectric bi-materials, respectively.

Substituting (7) and (8) into (5) and (6), we can obtain the dynamic antiplane governing equations for piezoelectric materials; as follows (Parton and Kudryavtsev, 1988):

$$c_{44}^{(i)}\nabla^2 w^{(i)}+e_{15}^{(i)}\nabla^2\phi^{(i)}=\rho_i\partial^2 w^{(i)}/\partial t^2 \tag{9}$$

$$e_{15}^{(i)}\nabla^2 w^{(i)}-\varepsilon_{11}^{(i)}\nabla^2\phi^{(i)}=0 \tag{10}$$

In the above equations, $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2$ is the two-dimensional Laplacian operator. It should be noted that the body force and the free charge are neglected in the present work.

Substituting Eq. (10) into Eq. (9), we can obtain the wave equation for piezoelectric materials, viz.

$$\nabla^2 w^{(i)}=c_2^{(i)^{-2}}\partial^2 w^{(i)}/\partial t^2 \tag{11}$$

in which

$$c_2^{(i)}=\sqrt{\mu^{(i)}/\rho_i} \tag{12}$$

being the antiplane shear wave speed in the piezoelectric materials, with

$$\mu^{(i)}=c_{44}^{(i)}+e_{15}^{(i)^2}/\varepsilon_{11}^{(i)}$$

3. Derivation of the singular integral equations

Eq. (11) has following solution in the Laplace transform domain with respect to time

$$w^{(i)*}(x,y,p)=\frac{2}{\pi}\int_0^\infty A_i(s,p)\exp[(-1)^i\gamma_i y]\cos(sx)\,ds \tag{13}$$

where

$$\begin{aligned} &\gamma_i(s,p) = \sqrt{s^2 + p^2 c_2^{(i)^{-2}}} \\ &w^{(i)*}(x,y,p) = \int_0^\infty w^{(i)}(x,y,t)\exp(-pt)\,\mathrm{d}t \\ &w^{(i)}(x,y,t) = \frac{1}{2\pi \mathrm{i}}\int_{\mathrm{Br}} w^{(i)*}(x,y,p)\exp(pt)\,\mathrm{d}p \end{aligned} \tag{14}$$

in which p denotes the Laplace transform parameter, Br stands for the Bromwich path of integration. The quantities with an asterisk denote the corresponding Laplace transforms. It is worth noting that the condition that the mechanical displacement should be bounded at infinity is incorporated in the derivation of Eq. (13). Inserting Eq. (13) into Eq. (10) leads to

$$\phi^{(i)*}(x,y,p) = \frac{e_{15}^{(i)}}{\varepsilon_{11}^{(i)}} w^{(i)*}(x,y,p) + \psi^{(i)*}(x,y,p) \tag{15}$$

where

$$\psi^{(i)*}(x,y,p) = \int_0^\infty \psi^{(i)}(x,y,t)\exp(pt)\,\mathrm{d}t = \frac{2}{\pi}\int_0^\infty B_i(s,p)\exp[(-1)^i sy]\cos(sx)\,\mathrm{d}s \tag{16}$$

Similarly, the condition that the electric potential should be bounded at infinity is incorporated in the derivation of (15). Substituting Eq. (15) into the Laplace transform of Eqs. (7) and (8), it follows that:

$$\tau_{zj}^{(i)*} = \mu^{(i)} w,_j^{(i)*} + e_{15}^{(i)} \psi,_j^{(i)*} \tag{17}$$

$$D_j^{(i)*} = -\varepsilon_{11}^{(i)} \psi,_j^{(i)*} \tag{18}$$

Substituting (13) and (16) into (17) and (18) results in

$$\tau_{zy}^{(i)*} = -\frac{2}{\pi}(-1)^i \int_0^\infty \{\mu^{(i)}\gamma_i A_i(s,p)\exp[(-1)^i\gamma_i y] + e_{15}^{(i)} s B_i(s,p)\exp[(-1)^i sy]\}\cos(sx)\,\mathrm{d}s \tag{19}$$

$$\tau_{zx}^{(i)*} = -\frac{2}{\pi}\int_0^\infty \{\mu^{(i)}\gamma_i A_i(s,p)\exp[(-1)^i\gamma_i y] + e_{15}^{(i)} s B_i(s,p)\exp[(-1)^i sy]\}\sin(sx)\,\mathrm{d}s \tag{20}$$

$$D_y^{(i)*} = -\frac{2}{\pi}(-1)^i \varepsilon_{11}^{(i)} \int_0^\infty s B_i(s,p)\exp[(-1)^i sy]\cos(sx)\,\mathrm{d}s \tag{21}$$

$$D_x^{(i)*} = -\frac{2}{\pi}\varepsilon_{11}^{(i)} \int_0^\infty s B_i(s,p)\exp[(-1)^i sy]\sin(sx)\,\mathrm{d}s \tag{22}$$

The Laplace transform of the boundary conditions yields

$$\left\{\begin{aligned} &\tau_{zy}^{(1)*}(x,0,p) = \tau_{zy}^{(2)*}(x,0,p), \\ &D_y^{(1)*}(x,0,p) = D_y^{(2)*}(x,0,p), \end{aligned}\right. \quad |x| < \infty \tag{23}$$

$$\phi^{(1)*}(x,0,p) = \phi^{(2)*}(x,0,p), \quad |x| < \infty \tag{24}$$

$$w^{(1)*}(x,0,p) = w^{(2)*}(x,0,p), \quad 0 < |x| < a, \ |x| > b \tag{25}$$

$$\tau_{zy}^{(1)*}(x,0,p) = \tau_{zy}^{(2)*}(x,0,p) = -\tau_0/p, \quad a < |x| < b \tag{26}$$

Substituting (19), (21), (13), (16) into the boundary conditions (23) and (24), we can obtain the relation between the four unknowns $A_\mathrm{i}(s,p)$ and $B_\mathrm{i}(s,p)$:

$$\varepsilon_{11}^{(1)} B_1(s,p) = -\varepsilon_{11}^{(2)} B_2(s,p) \tag{27}$$

$$-\mu^{(1)}\gamma_1 A_1(s,p) - e_{15}^{(1)} s B_1(s,p) = \mu^{(2)}\gamma_2 A_2(s,p) + e_{15}^{(2)} s B_2(s,p) \tag{28}$$

$$\frac{e_{15}^{(1)}}{\varepsilon_{11}^{(1)}} A_1(s,p) + B_1(s,p) = \frac{e_{15}^{(2)}}{\varepsilon_{11}^{(2)}} A_2(s,p) + B_2(s,p) \tag{29}$$

From the above relations, it can be realized that the four unknowns $A_i(s,p)$ and $B_i(s,p)$, $(i = 1,2)$ are not independent of each other. Hence, we can express the four unknowns in terms of one of them, for example, $A_2(s,p)$:

$$A_1(s,p) = a_1(s,p) A_2(s,p) \tag{30}$$

$$B_2(s,p) = b_2(s,p) A_2(s,p) \tag{31}$$

$$B_1(s,p) = b_1(s,p) A_2(s,p) \tag{32}$$

in which

$$a_1(s,p) = \frac{\varepsilon_{11}^{(1)} e_{15}^{(2)}}{\varepsilon_{11}^{(2)} e_{15}^{(1)}} + \left(1 + \frac{\varepsilon_{11}^{(2)}}{\varepsilon_{11}^{(1)}}\right) \frac{\varepsilon_{11}^{(1)}}{e_{15}^{(1)}} b_2(s,p) \tag{33}$$

$$b_1(s,p) = -\frac{\varepsilon_{11}^{(2)}}{\varepsilon_{11}^{(1)}} b_2(s,p) \tag{34}$$

$$b_2(s,p) = \left(\mu^{(2)}\gamma_2 + \mu^{(1)}\gamma_1 \frac{\varepsilon_{11}^{(1)} e_{15}^{(2)}}{\varepsilon_{11}^{(2)} e_{15}^{(2)}}\right) \left[\left(\frac{e_{15}^{(1)} \varepsilon_{11}^{(2)}}{\varepsilon_{11}^{(1)}} - e_{15}^{(2)}\right) s - \mu^{(1)}\gamma_1 \left(\varepsilon_{11}^{(1)} + \varepsilon_{11}^{(2)}\right) \Big/ e_{15}^{(1)}\right]^{-1} \tag{35}$$

Substituting (19), (13) and (16) into (25) and (26), and accounting for the relations of Eqs. (33)–(35), leads to the dual integral equation in terms of $A_2(s,p)$, as follows:

$$\frac{2}{\pi}\int_0^\infty s f(s,p) A_2(s,p)\cos(sx)\,\mathrm{d}s = -\tau_0/(p\mu^{(2)}), \quad a < x < b \tag{36}$$

$$\frac{2}{\pi}\int_0^\infty A_2(s,p)\cos(sx)\,\mathrm{d}s = 0, \quad 0 < x < a, \;\; x > b \tag{37}$$

where

$$f(s,p) = \gamma_2/s + e_{15}^{(2)} b_2(s,p)/\mu^{(2)} \tag{38}$$

Let us now introduce a new unknown function $g(u,p)$ along the crack surface, such that

$$A_2(s,p) = s^{-1}\int_a^b g(u,p)\sin(su)\,\mathrm{d}u \tag{39}$$

Substituting Eq. (39) into Eqs. (36) and (37), using the corresponding Fourier integral formula (Erdelyi et al., 1954), we can obtain the Cauchy-type singular integral equation of the first kind for $g(u,p)$, viz.

$$\begin{aligned} &\frac{1}{\pi}\int_a^b \left[\frac{\bar{f}}{u-x} + k(u,x,p)\right] g(u,p)\,\mathrm{d}u = -\tau_0/\left(p\mu^{(2)}\right) \\ &\frac{1}{\pi}\int_a^b g(u,p)\,\mathrm{d}u = 0 \end{aligned} \tag{40}$$

in which the kernel is expressed by

$$k(u,x,p)=\frac{\bar{f}}{u+x}+2\int_0^\infty [f(s,p)-\bar{f}]\sin(su)\cos(sx)\,\mathrm{d}s \tag{41}$$

with

$$\bar{f}=\lim_{s\to\infty} f(s,p)=1+e_{15}^{(2)}\left(1+\frac{\mu^{(1)}\varepsilon_{11}^{(1)}e_{15}^{(2)}}{\mu^{(2)}\varepsilon_{11}^{(2)}e_{15}^{(2)}}\right)\Big/\left[e_{15}^{(1)}\varepsilon_{11}^{(2)}/\varepsilon_{11}^{(1)}-e_{15}^{(2)}-\mu^{(1)}\left(\varepsilon_{11}^{(1)}+\varepsilon_{11}^{(2)}\right)/e_{15}^{(1)}\right] \tag{42}$$

Using the following transform of variables:

$$x=\frac{b+a}{2}+\frac{b-a}{2}r,\quad u=\frac{b+a}{2}+\frac{b-a}{2}q \tag{43}$$

the singular integral equation (40) can be recast in the following form:

$$\begin{aligned}&\frac{1}{\pi}\int_{-1}^{1}\left[\frac{\bar{f}}{q-r}+k_s(q,r,p)\right]G(q,p)\,\mathrm{d}q=-\tau_0\big/\left(p\mu^{(2)}\right)\\&\frac{1}{\pi}\int_{-1}^{1}G(q,p)\,\mathrm{d}q=0\end{aligned} \tag{44}$$

with

$$\begin{aligned}&G(q,p)=g\left(\frac{b+a}{2}+\frac{b-a}{2}q,p\right)\\&k_s(q,r,p)=\frac{b-a}{2}k\left(\frac{b+a}{2}+\frac{b-a}{2}q,\frac{b+a}{2}+\frac{b-a}{2}r,p\right)\end{aligned} \tag{45}$$

Considering the square-root singularity of the electro-mechanical fields around the tip of an interfacial crack, we can assume that:

$$G(q,p)=\frac{-\tau_0}{\mu^{(2)}p}G_s(q,p)(1-q^2)^{-1/2} \tag{46}$$

Applying the Gauss–Chebyshev formulation (Erdogan and Gupta, 1972) to Eq. (44), we can obtain the following algebraic equation system:

$$\begin{aligned}&\sum_{i=1}^{n}\frac{1}{n}G_s(q_i,p)\left[\frac{\bar{f}}{q_i-r_j}+k_s(q_i,r_j,p)\right]=1\\&\sum_{i=1}^{n}\frac{1}{n}G_s(q_i,p)=0\end{aligned} \tag{47}$$

where

$$q_i=\cos\left(\frac{2\mathrm{i}-1}{2n}\pi\right),\qquad r_j=\cos\left(\frac{j}{n}\pi\right)\quad j=1,2,\ldots,n-1 \tag{48}$$

The antiplane dynamic stress intensity factor of the interfacial crack $K_3(t)$ is defined by

$$\begin{bmatrix}K_3^a(t)\\K_3^b(t)\end{bmatrix}=\begin{bmatrix}\lim\limits_{x\to a^-}\\\lim\limits_{x\to b^+}\end{bmatrix}\begin{bmatrix}\sqrt{2\pi(a-x)}\\\sqrt{2\pi(x-b)}\end{bmatrix}\tau_{zy}^2(x,0,t) \tag{49}$$

Then, from Eqs. (19), (39), (49), with s approaching infinity (Fan, 1990), we find the following singular behaviour at the tip of the interfacial crack:

$$K_3^{a,b}(t)/(\tau_0\sqrt{\pi(b-a)/2}) = \frac{\mp\bar{f}}{2\pi \mathrm{i}}\int_{\mathrm{Br}}[G_s(\mp 1,p)/p]\exp(pt)\,\mathrm{d}p \tag{50}$$

Consequently, the antiplane stress field around the tip of the interfacial cracks can be expressed in terms of the stress intensity factor as being:

$$\begin{cases} \tau_{zx}(r,\theta,t) = \dfrac{K_3(t)}{\sqrt{2\pi r}}\sin\dfrac{\theta}{2} \\ \tau_{zy}(r,\theta,t) = \dfrac{K_3(t)}{\sqrt{2\pi r}}\cos\dfrac{\theta}{2} \end{cases} \tag{51}$$

in which (r,θ) are the polar coordinates around the crack tips (Fig. 1).

4. Numerical results and discussions

Solving the algebraic equation system (47), and accomplishing the Laplace inversion of (50), the antiplane dynamic stress intensity factor (DSIF) can be obtained for different combinations of piezoelectric bi-materials. The Laplace inversion was carried out using the method of Miller and Guy (1966) (see

Table 1
Material properties of commonly used piezoelectric ceramics (Parton and Kudryavtsev, 1988)

	ρ ($\times 10^3$) kg/m^3	ε_{11} ($\times 10^{-9}$) F/m	c_{44} ($\times 10^{10}$) N/m^2	e_{15}, C/m	μ ($\times 10^{10}$) N/m^2	c_2 ($\times 10^3$) m/s
PZT-4	7.5	6.4634	2.56	12.7	5.055	2.6
BaTiO3	5.7	9.8722	4.4	11.4	5.716	3.17
PZT 65/35	7.825	5.66	3.89	8.387	5.133	2.56
ZnO	5.68	0.0757	4.247	−0.48	4.551	2.83

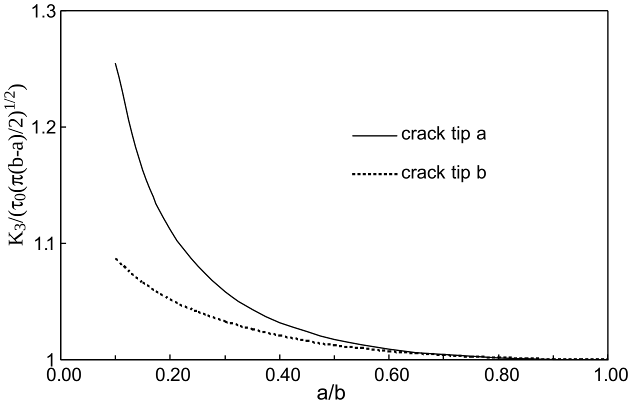

Fig. 2. Static stress intensity factor of two coplanar interfacial cracks versus a/b under antiplane loading.

Appendix A for details). In the calculation, the stress intensity factor was normalized as $k_3(t)/[\tau_0\sqrt{\pi(b-a)/2}]$.

The following five combinations of piezoelectric bi-materials, commonly used in smart structures, were selected in our computation:

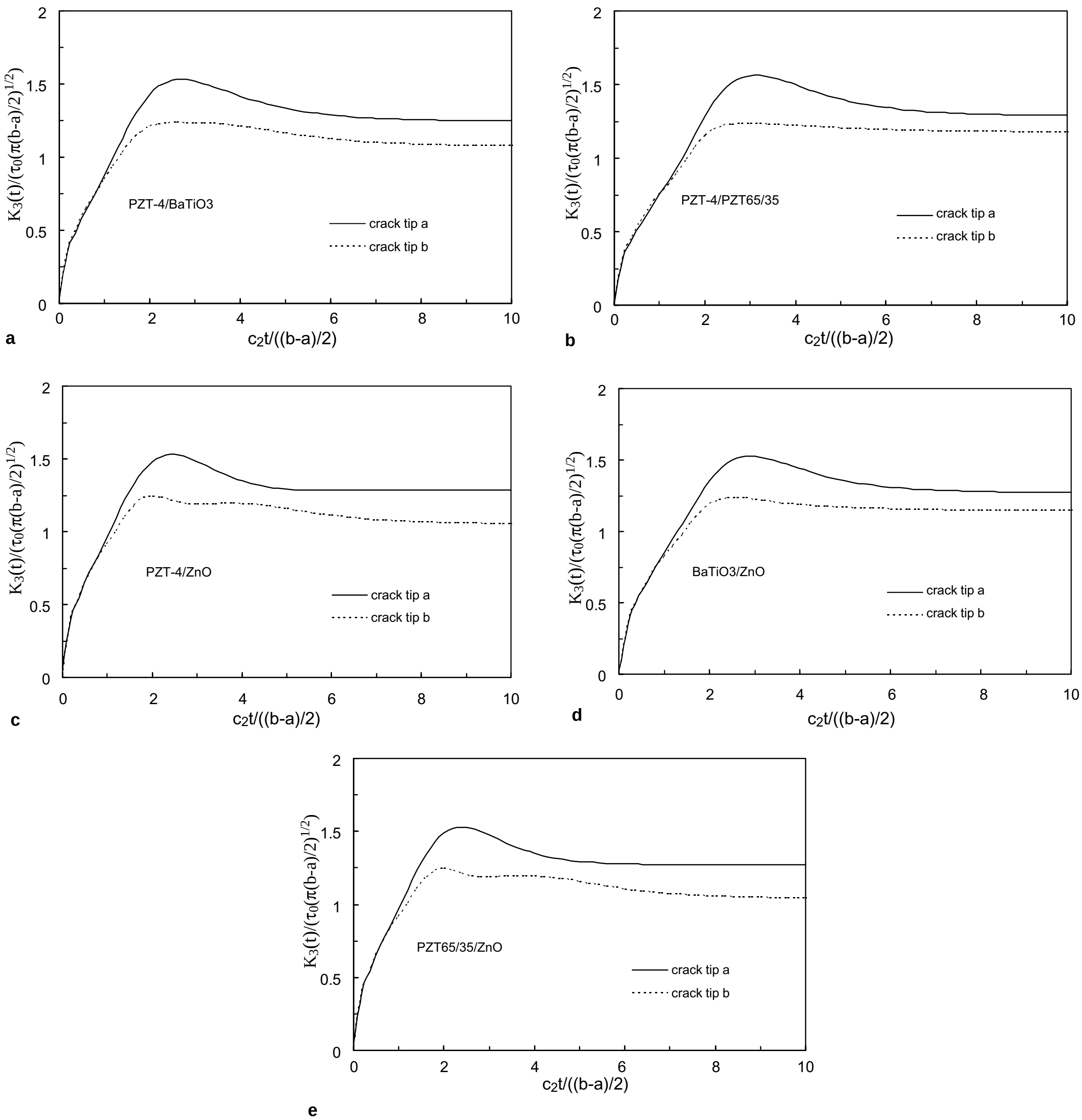

Fig. 3. Dynamic stress intensity factors against normalized time for different piezoelectric bi-materials: (a) PZT-4/BatiO$_3$; (b) PZT-4/PZT 65/35; (c) PZT-4/ZnO; (d) BatiO$_3$/ZnO; (e) PZT 65/35/ZnO.

(1) material I (upper): PZT-4, material II (lower): $BatiO_3$;
(2) material I (upper): PZT-4, material II (lower): PZT 65/35;
(3) material I (upper): PZT-4, material II (lower): ZnO;
(4) material I (upper): $BatiO_3$, material II (lower): ZnO;
(5) material I (upper): PZT 65/35, material II (lower): ZnO.

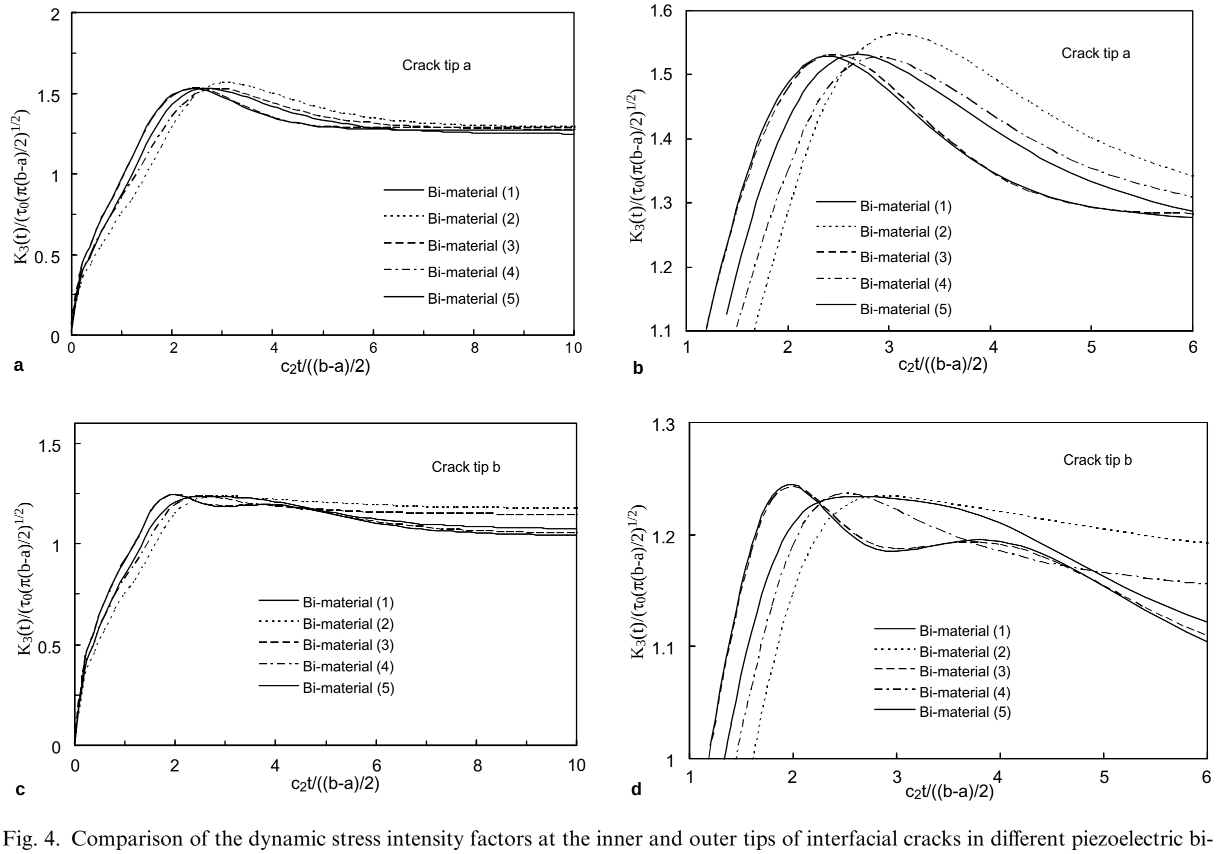

Fig. 4. Comparison of the dynamic stress intensity factors at the inner and outer tips of interfacial cracks in different piezoelectric bi-materials: (a) and (b) inner crack tips, and (c) and (d) outer crack tips.

Table 2
The peak value of DSIF and the corresponding normalized time for $a/b = 0.1$

Bi-materials	$\frac{k_3^a(t)}{\tau_0\sqrt{\pi(b-a)/2}}$	$c_2t/((b-a)/2)$	$\frac{k_3^b(t)}{\tau_0\sqrt{\pi}(b-a)/2}$	$c_2t/((b-a)/2)$
(1)	1.530	2.6	1.234	2.6
(2)	1.562	3.2	1.234	2.8
(3)	1.530	2.4	1.243	1.8
(4)	1.526	3.0	1.236	2.6
(5)	1.528	2.4	1.244	2.0

Static SIFs: $K_3^a/[\tau_0\sqrt{\pi(b-a)/2}] = 1.3198$, $K_3^b/[\sqrt{\pi(b-a)/2}] = 1.1465$.

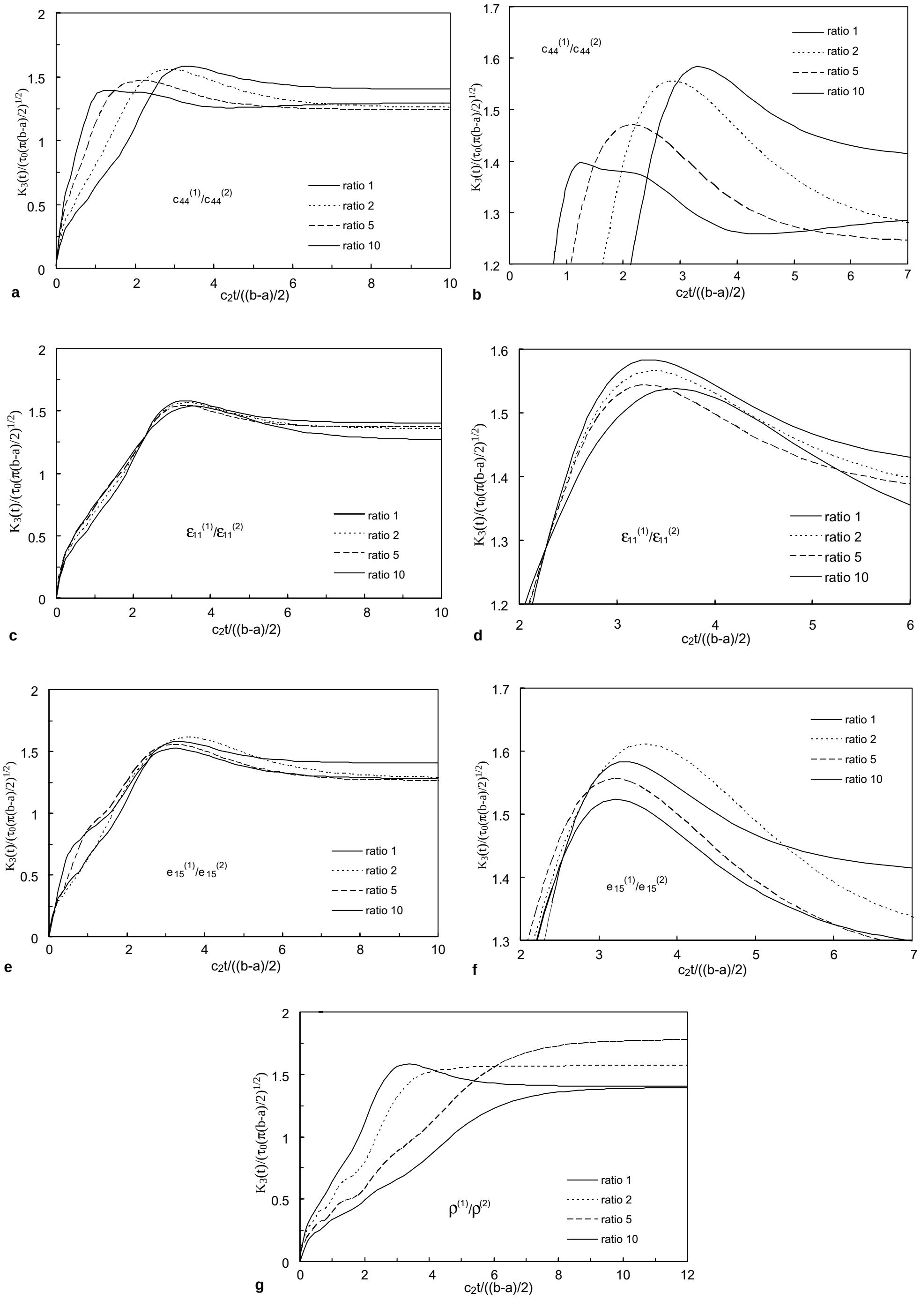

Fig. 5. Comparison of the influence of different material properties: bi-materials have the same values of all other material properties except: (a) and (b) c_{44}; (c) and (d) ε_{11}; (e) and (f) e_{15}; and (g) ρ.

The electro-mechanical properties of the above piezoelectric materials are listed in Table 1.

Fig. 2 shows the static SIF versus a/b, with a/b being the ratio of the distance between the inner and the outer tips of the coplanar interfacial cracks. The figure reveals that the SIFs at both the inner tip a and the outer tip b increase rapidly as the two cracks tend to coalesce. These interacting effects in the SIF become very significant as a/b approaches naught. It is found from Fig. 2 that the SIF at the inner tip a is always greater than that at the outer tip b, and that the difference between these two SIFs increases as a/b decreases. It is worth noting that in the static case, the SIF of the coplanar interface cracks is independent of the materials' properties, which recovers the conclusion for the single interface crack problem of piezoelectric bi-materials (Chen et al., 1997). As a/b approaches unity, the SIFs at both tips tend to unity. In this case, the interacting effect of the two cracks is very weak, and, therefore, the resulting SIFs at both tips approach that of a single interface crack (Chen et al., 1997).

Fig. 3(a)–(e) show the variation of the dynamic SIFs of the interface cracks in the five different types of piezoelectric bi-materials against the normalized time, $c_2t/((b-a)/2)$. In this study, a/b was selected to be 0.1 to ensure that interaction effects exist between these interface cracks. All dynamic stress intensity factors (DSIFs) in these figures show variations similar to those observed in the coplanar crack problem of homogenous elastic media (Itou, 1980) as well as piezoelectric materials (Chen and Meguid, 2000). The DSIF increases rapidly from zero to a peak value well above its corresponding static value, and then oscillates about it. In all the materials combinations considered, the DSIF gains the peak value at the normalized time $c_2t/((b-a)/2)$ of about 1.21–1.52. To compare the difference amongst the DSIFs of the five piezoelectric bi-materials, we present the variation of the DSIFs at the inner crack tip a in Fig. 4(a) and (b). These two figures indicate that the peak value of the DSIF varies in a very small range among the different combinations considered, although the normalized time when DSIF attains the peak value differs significantly. Fig. 4(c) and (d) show the variation of the DSIF at the outer tip b of the interface cracks in the different piezoelectric bi-materials considered. It is worth pointing out that Fig. 4(c) and (d) show similar dependence of DSIF upon the material combination, as outlined earlier in Fig. 4(a) and (b). The peak values of DSIFs and the corresponding normalized time, when DSIFs gain their peak values, are provided in Table 2 for the case when $a/b = 0.1$.

Fig. 5 shows the effects of each of the piezoelectric material properties on the DSIF. All other properties were held constant in the calculations except one as shown in Fig. 5(a) and (b) (c_{44}), Fig. 5(c) and (d) (ε_{11}), Fig. 5(e) and (f) (e_{15}), and Fig. 5(g) (density). Four different values, 1, 2, 5, and 10, of the ratio of the respective material property of material I to material II, were chosen to examine their effect on the dynamic response. It is clearly seen that increase in the ratio of $c_{44}^{(1)}/c_{44}^{(2)}$ induces a monotonic decrease in the predicted peak value of dynamic stress intensity factor and the corresponding time at which this peak value is achieved. For example, when the ratio of $c_{44}^{(1)}/c_{44}^{(2)}$ increases from 1 to 10, the predicted peak value of DSIF decreases from 1.58 at $c_2t/((b-a)/2) = 3.4$, to 1.39 at $c_2t/((b-a)/2) = 1.2$. The ratio of $\varepsilon_{11}^{(1)}/\varepsilon_{11}^{(2)}$ induces a milder influence on the predicted DSIF compared with $c_{44}^{(1)}/c_{44}^{(2)}$. Although variation in $e_{15}^{(1)}/e_{15}^{(2)}$ or $\rho^{(1)}/\rho^{(2)}$ strongly impacts the predicted DSIF, their effects are somewhat random, and therefore, difficult to describe, as shown in Fig. 5(e)–(g).

5. Conclusion

In this article, we investigate the dynamic fracture behaviour of two interface cracks in piezoelectric bi-materials. Integral transform technique is employed to reduce the current initial–boundary-value problem to the solution of singular integral equation, which is then solved by the aid of a collocation method. The resulting dynamic stress intensity factors are obtained by performing Laplace inversion. Five kinds of commonly used piezoelectric bi-materials are considered in the numerical calculation to illustrate the effect of the presence of interacting interfacial cracks and material combinations upon the impact response of cracked piezoelectric bi-materials. It is revealed that:

(i) the quasi-static SIF of the interfacial cracks in piezoelectric bi-materials is independent of the material properties of the piezoelectric bi-materials,
(ii) the variation of the DSIF is similar to that in pure elastic solids as well as homogeneous piezoelectric materials, although both the peak value and the normalized time when DSIF gains its peak value depends upon the properties of the piezoelectric bi-materials, and
(iii) the DSIF at the inner crack tip is always greater than that at the outer crack tip, indicating that the coplanar cracks tend to coalesce at first, and then propagate from the outer tips.
(iv) The ratios of elastic modulus and mass density of the two materials greatly affects the predicted dynamic response of the piezoelectric bi-materials. Other material parameters, such as piezoelectric and dielectric materials, play secondary roles in the dynamic fracture response of the piezoectric bi-materials.

Appendix A. Numerical inversion of Laplace transform

According to Miller and Guy (1966), see also (Chen and Sih, 1977), when Laplace's transform of a function of time $f(t)$ is $f^*(p)$, then it can be evaluated at discrete points given by

$$p = (\beta + 1 + n)\delta, \quad n = 1, 2, \ldots \tag{A.1}$$

Accordingly, we can determine the coefficients C_m from the following set of equations:

$$\delta f^*[(\beta + 1 + n)\delta] = \sum_{m=0}^{n} \frac{C_m n!}{(\beta + n + 1)(\beta + n + 2) \cdots (\beta + n + 1 + m)(n - m)!} \tag{A.2}$$

where $\delta > 0$ and $\beta > -1.0$. If the coefficients are calculated up to C_{N-1}, an approximation of $f(t)$ can be found as

$$f(t) = \sum_{m=0}^{N-1} C_m P_m^{(0,\beta)}[2\exp(-\delta t) - 1] \tag{A.3}$$

where $P_m^{(0,\beta)}(x)$ is a Jacobi polynomial and N is the number of terms employed. The parameters β, δ and N are selected such that $f(t)$ can be best described within a particular period of time t. In practice, we can select $\beta = 0$, then the Jacobi polynomial becomes Legendre polynomial (Chen and Sih, 1977).

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